Answer of HW1

•Equilibrium conditions:

In the vertical direction (Fig.1)

$$P = N_2 + 2N_1 \sin\alpha \qquad (1)$$

•Compatibility conditions:

From Fig.2,

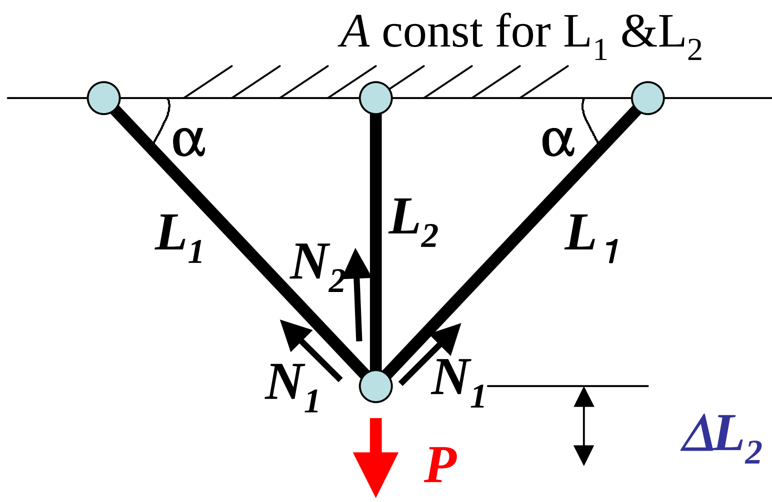
$$\frac{\Delta L_1}{\sin(\alpha + \delta\alpha/2)} = \frac{\Delta L2}{\cos(\delta\alpha/2)} \qquad (2)$$

Since ΔL_2 is very small compared to L_2, $\delta\alpha/2$ is also very small so that

$$\sin(\alpha + \delta\alpha/2) \approx \sin\alpha, \quad \cos(\delta\alpha/2) \approx 1$$

Hence,

$$\Delta L_1 = \Delta L_2 \sin\alpha \quad (\prec \Delta L_2) \qquad (3)$$



N_i: axial force of truss member

A: cross sectional area

Fig.1

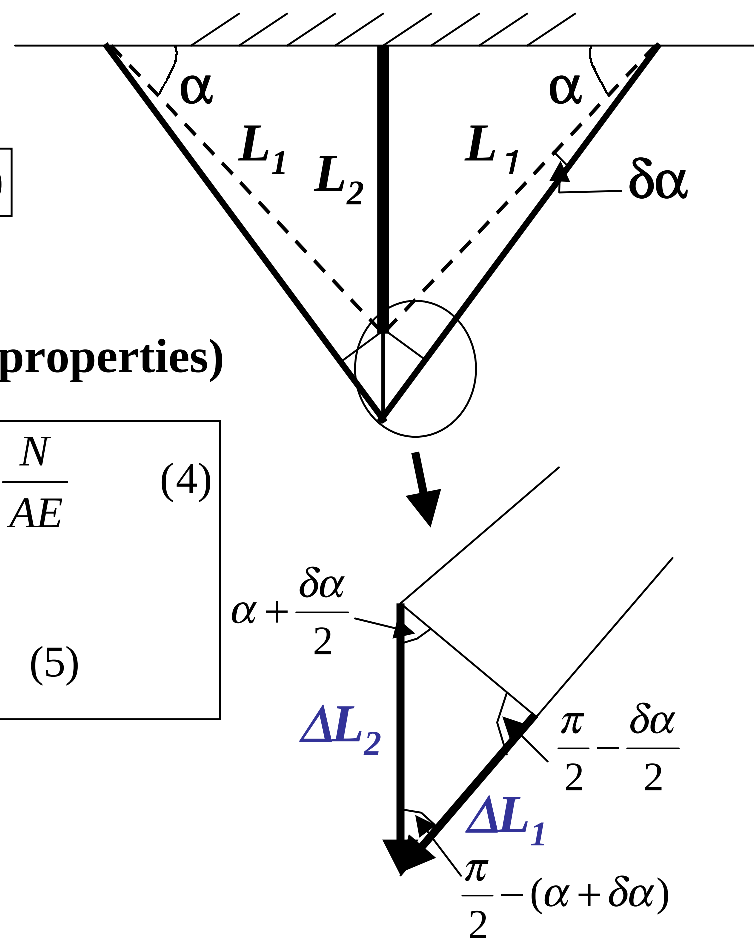


Fig.2

•Constitutive relation (material properties) (Fig.3)

Before yielding: (Hooke's law)

$$\varepsilon = \frac{\Delta L}{L} = \frac{\sigma}{E} = \frac{N}{AE} \qquad (4)$$

After yielding:

$$\sigma = \frac{N}{A} = Y \qquad (5)$$

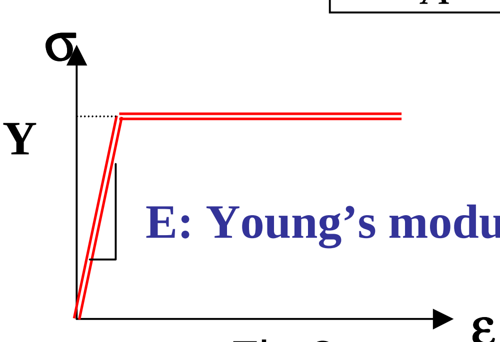


Fig.3

Load(P) – vertical displacement (DL_2)relation

- In the elastic range for both L_1 and L_2:

From eq(4),

$$\Delta L_1 = \frac{L_1 N_1}{AE} \quad (6-1), \qquad \Delta L_2 = \frac{L_2 N_2}{AE} \quad (6-2)$$

and

$$N_1 = \frac{AE}{L_1}\Delta L_1 \quad (7-1), \qquad N_2 = \frac{AE}{L_2}\Delta L_2 \quad (7-2)$$

Substituting eqs. (7) into eq(1),

•Equilibrium cond.
•Constitutive relation
•Compatibility

$$P = \frac{AE}{L_2}\Delta L_2 + 2\sin\alpha \frac{AE}{L_1}\Delta L_1 \qquad (8)$$

From eq(8) and eq(3), and $L_2/L_1=\sin\alpha$

$$P = \frac{AE}{L_2}(1+2\sin^3\alpha)\Delta L_2 \qquad (9)$$

(b) In the range where L_1 and L_2 are plastic and elastic respectively:

From eq(5), N_2 after yielding,

$$N_2 = AY \qquad (10)$$

Since L_1 is still elastic range, eq(7-1) is valid for L_1.

Hence by substituting eqs(7-1) and (10) into eq(1) and using eq(3) and $L_2/L_1=\sin\alpha$,

•Constitutive relation

$$P = AY + \frac{2AE}{L_2}\sin^3\alpha\Delta L_2 \qquad (11)$$

At the load P where $N_2/A=E\Delta L_2/L_2=Y$, L_2 reaches plastic conditions.

Hence at this load, $\Delta L_2 = YL_2/E$ (12)

(c) In the range where L_1 and L_2 are both plastic condition:

Since $N_1 = AY$ (13)

From eqs (10) & (13) and eq(1)

$$P = AY(1 + 2\sin\alpha) \qquad (14)$$

At the load P where $N_1/A = E\Delta L_1/L_2 = Y$, L_1 reaches plastic conditions.

At this load, $\Delta L_1 = YL_1/E$ (15)

From eq(13), eq(3) and $L_2/L_1 = \sin\alpha$,

ΔL_2 at this load,

$$\Delta L_2 = \frac{YL_2}{E\sin^2\alpha} \qquad (16)$$